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Branch-Line Couplers Using Unequal Line Lengths

Canan Toker, Mustafa Saglam, Mustafa Ozme, and Nilgun Gunalp

***Abstract*—General solutions for branch-line couplers using different line lengths are provided in this paper. Explicit expressions are derived with which a hybrid with any given power division ratio can be designed. In the design, one of the characteristic impedances or lengths of the branches can be chosen arbitrarily to suit a given design specification. This approach brings flexibility in choosing the characteristic impedances or the lengths of the branches, and is helpful especially in monolithic-microwave integrated-circuit applications where restrictions imposed on microstrip transmission lines do not allow the use of conventional branch-line couplers employing quarter-wave-long lines. However, the resultant bandwidth is narrower compared to that of the conventional coupler.**

***Index Terms*—Branch-line couplers, hybrids.**

I. INTRODUCTION

Ever since the introduction of the branch-line couplers, it has been a common practice to choose all the branches a quarter-wave-long at the center of the frequency band of interest. With fixed lengths of the lines, attention is then concentrated on the characteristic impedances of the various branches to satisfy the given specifications. This procedure is adopted for all the work on branch line or branch-guide couplers, including multibranch couplers, which are developed to widen the inherently narrow bandwidths [1], [2]. The problem of realizability concerning the characteristic impedances of the multibranch microstrip couplers is improved by optimization algorithms; however, in such studies, the lengths are again kept fixed and only the characteristic impedances are optimized [3]. Broad-band uniplanar branch-line couplers, which are more suitable for microwave-integrated-circuit (MIC) and monolithic-microwave integrated-circuit (MMIC) applications also use quarter-wave-long lines [4].

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C. Toker and N. Gunalp are with the Department of Electrical and Electronics Engineering, Middle East Technical University, 06531 Ankara, Turkey.

M. Saglam was with MIKES (Microwave Electronic Systems Inc.), 06531 Ankara, Turkey. He is now with the Institute of Microwave Electronics, Darmstadt University of Technology, 64283 Darmstadt, Germany.

M. Ozme is with Military Electronics Industries, ASELSAN (Military Electronics Industries), 06531 Ankara, Turkey.

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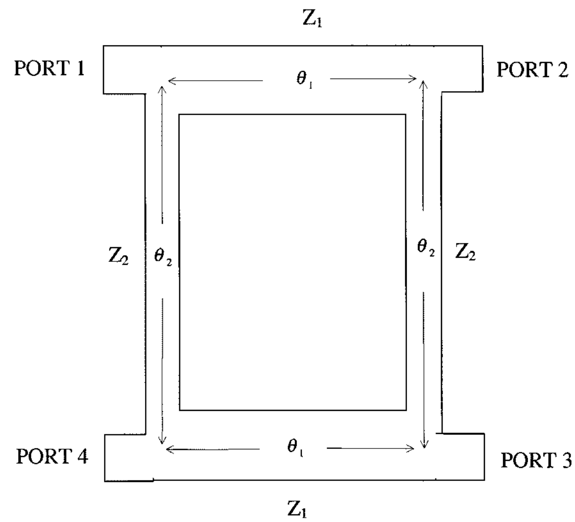

Fig. 1. Unequal line-length branch-line coupler.

Branch-line couplers with quarter-wave-long lines being satisfactory for most applications, similar constructions, but having different line lengths, are the interest of the work presented in this paper, which has not been considered previously. It is not only interesting from the academic point-of-view, but it is also useful, for some applications, to use this alternative design because the hybrids using quarter-wave-long lines do not always lead to realizable characteristic impedances. This fact is obvious in some MIC and MMIC applications where the values of the characteristic impedances of the microstrip transmission lines are outside the realizable range. Varying the lengths of the branches introduces additional freedom in choosing the characteristic impedances to overcome this realizability problem, as will be discussed below.

In the alternative design presented in this paper (see Fig. 1), the characteristic impedances and lengths of the through and branch lines are different. This type of construction enables one to deal with four variables, namely, two characteristic impedances and two lengths compared to the conventional coupler where only two characteristic impedances are used as variables. The analysis of the new branch-line coupler leads to three design equations with which a hybrid with any power division ratio can be designed readily. In this design, one of the parameters can be chosen arbitrarily, which may be very useful under certain circumstances, and the phase angle difference between the transmitted signals to the output branches is 90°. However, bandwidth reduction is unavoidable.

II. ANALYSIS

The branch-line coupler under investigation is shown in Fig. 1. This coupler has different line lengths and characteristic impedances for the adjacent arms. The lines are assumed to be lossless and reciprocal, and the reference impedance at the ports is taken as Z_0. Z_1 is the characteristic impedance of the through lines and Z_2 is the characteristic impedance of the branch lines, θ_1 and θ_2 being their respective electrical lengths. Port 1 is the input port and port 4 is the isolated port. Ports 2 and 3 are the output ports. Applying even- and odd-mode analysis [5], the chain parameters are obtained as follows:

For even-mode excitation from ports 1 and 4

$$A^e = D^e = \cos\theta_1 - \frac{Z_1}{Z_2}\tan\frac{\theta_2}{2}\sin\theta_1 \tag{1}$$

$$B^e = jZ_1\sin\theta_1 \tag{2}$$

$$C^e = 2j\frac{1}{Z_2}\tan\frac{\theta_2}{2}\cos\theta_1 + j\frac{1}{Z_2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\tan^2\frac{\theta_2}{2}\right)\sin\theta_1. \tag{3}$$

For odd-mode excitation from ports 1 and 4

$$A^o = D^o = \cos\theta_1 + \frac{Z_1\sin\theta_1}{Z_2\sin\frac{\theta_2}{2}} \tag{4}$$

$$B^o = jZ_1\sin\theta_1 \tag{5}$$

$$C^o = -2j\frac{1}{Z_2}\frac{\cos\theta_1}{\tan\frac{\theta_2}{2}} + j\frac{1}{Z_2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\frac{1}{\tan^2\frac{\theta_2}{2}}\right)\sin\theta_1. \tag{6}$$

For the even mode, the input reflection coefficient and the transmission coefficient from port 1 to port 2 are given by

$$\Gamma^e = \frac{\frac{B^e}{Z_0} - Z_0C^e}{2A^e + \frac{B^e}{Z_0} + Z_0C^e} \tag{7}$$

$$T_{21}^e = \frac{2}{2A^e + \frac{B^e}{Z_0} + Z_0C^e}. \tag{8}$$

Similarly, the expressions are obtained for the odd-mode coefficients by replacing the even-mode chain parameters by the odd-mode parameters in (7) and (8). After the superposition of even- and odd-mode excitations, the matching condition at the input ($S_{11} = 0$) and the isolation condition at port 4 ($S_{41} = 0$) are imposed. These conditions dictate that $\Gamma^e = \Gamma^o = 0$, resulting in

$$\frac{B^e}{Z_0} = Z_0C^e \tag{9}$$

$$C^e = C^o. \tag{10}$$

Hence, it is interesting to note that, for this structure not only $B^e = B^o$, but also $C^e = C^o$.

Equating (3) and (6) in accordance to (10), and simplifying the resultant expression, one finally obtains

$$Z_1\tan\theta_1 = -Z_2\tan\theta_2. \tag{11}$$

This is one of the three equations that are used in the design of the coupler. If one of the arms is chosen as $\theta_1 = 90^\circ$, (11) gives that θ_2 must also be selected as 90°, which is the case in the conventional branch-line coupler. However, (11) shows that other solutions exist.

The second design equation is obtained from (9). Substituting (2) and (3) into (9), an expression is obtained that can be simplified by invoking the condition given by (11) and several trigonometric identities. The resultant equation is

$$\frac{1}{Z_1^2} - \frac{1}{Z_2^2} = \frac{1}{Z_0^2}. \tag{12}$$

Note that when $Z_2 = Z_0$, this equation leads to the condition $Z_1 = Z_0/\sqrt{2}$, which is the case for a 3-dB conventional coupler. However, an infinite number of solutions exist for Z_1 and Z_2 provided that (11), (12), and the following results are satisfied.

Substituting the relevant chain parameters into the transmission coefficient expressions, invoking the condition given by (11), and several trigonometric identities, the ratio of signals traveling through ports 2 and 3 can be formed as

$$\frac{S_{21}}{S_{31}} = j\frac{Z_2}{Z_0}\sin\theta_2. \tag{13}$$

Equation (13) verifies that the signal through port 2 leads the signal through port 3 by 90° for all values of Z_2 and θ_2 provided that (11) and (12) are satisfied.

Defining the power division ratio k^2 between the output ports as *(power output through port 2)* $= k^2\times$ *(power input to port 1)*, where $0 < k < 1$, one can write from the conservation of power

$$\left|\frac{S_{21}}{S_{31}}\right|^2 = \frac{k^2}{1-k^2}. \tag{14}$$

Using (13) and (14)

$$Z_2|\sin\theta_2| = Z_0\frac{k}{\sqrt{1-k^2}} \tag{15}$$

is obtained. This relationship governs the power division between the output ports of the coupler and constitutes the third design equation.

III. Design Considerations

The design equations (11), (12), and (15) contain the four variables Z_1, Z_2, θ_1, and θ_2, one of which can be chosen arbitrarily. This choice can be made to suit the realizability constraints of the coupler. Usually, Z_1 is difficult, in practice, to implement and, hence, it would be useful to choose Z_1 as a realizable value in the beginning of the design.

From (12) and (15), it is readily observed that, for all designs, $Z_1 < Z_0$, $Z_2 > Z_1$, and $Z_2 \geq Z_0k/\sqrt{1-k^2}$. In a design procedure, k is first determined for a given power division ratio. Then, choosing one of the four variables independently, the others are obtained using (11), (12), and (15).

For a nominal 3-dB coupler, $k = 1/\sqrt{2}$. Substituting this value in (15), we obtain

$$Z_2\sin\theta_2 = Z_0. \tag{16}$$

For a conventional branch-line coupler with $\theta_2 = 90^\circ$, (16) gives $Z_2 = Z_0$, and (12) gives $Z_1 = Z_0/\sqrt{2}$, as expected.

If one preselects Z_1, the first equation to use would be (12) to find Z_2. θ_2 is then calculated from (16). Finally, θ_1 is evaluated using (11). As an example, for a 3-dB hybrid with $Z_0 = 50\ \Omega$, let us choose $Z_1 = 40\ \Omega$. This value is smaller than Z_0 as required, but larger than 35.35 Ω, which corresponds to Z_1 of a conventional coupler. Note that in some MMIC applications, the realization of a 35.35-Ω line is not possible. Corresponding Z_2 is $Z_2 = 66.67\ \Omega$. Using (16), two values are then obtained for θ_2 as $\theta_2 = 48.59^\circ$ or $\theta_2 = 131.41^\circ$. Finally, θ_1 is obtained from (11) as $\theta_1 = 117.89^\circ$ (corresponding to $\theta_2 = 48.59^\circ$) or $\theta_1 = 62.11^\circ$ (corresponding to $\theta_2 = 131.41^\circ$).

If the highest realizable value for Z_2 is taken as 100 Ω, still a higher value for Z_1 can be obtained as follows: choosing $Z_2 = 100\ \Omega$, from (12), $Z_1 = 44.72\ \Omega$, from (16), $\theta_2 = 30^\circ$ (or 150°), and from (11), $\theta_1 = 127.76^\circ$ (or 52.24°) are obtained.

These examples show that the branch-line coupler using unequal branch lengths reduces, to some extent, the problem of linewidths be-

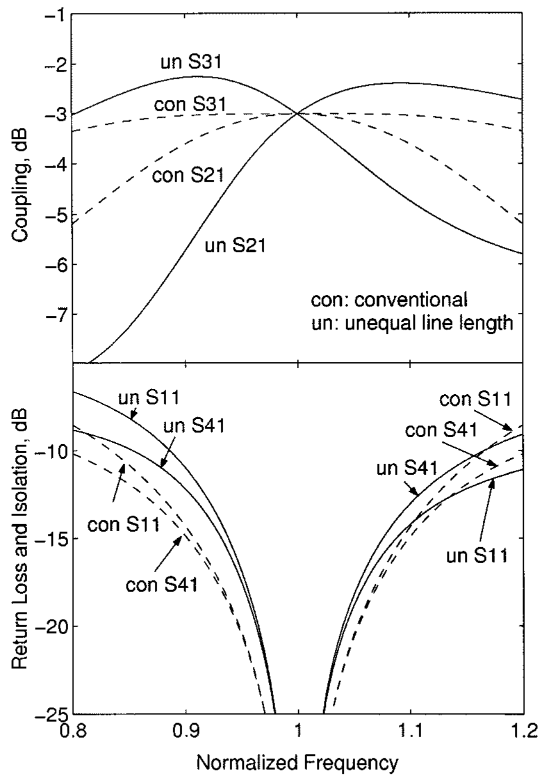

Fig. 2. Comparison of the frequency responses of a 3-dB unequal line length and a conventional coupler. $Z_1 = 40\ \Omega$, $Z_2 = 66.67\ \Omega$, $\theta_1 = 117.89°$, and $\theta_2 = 48.59°$.

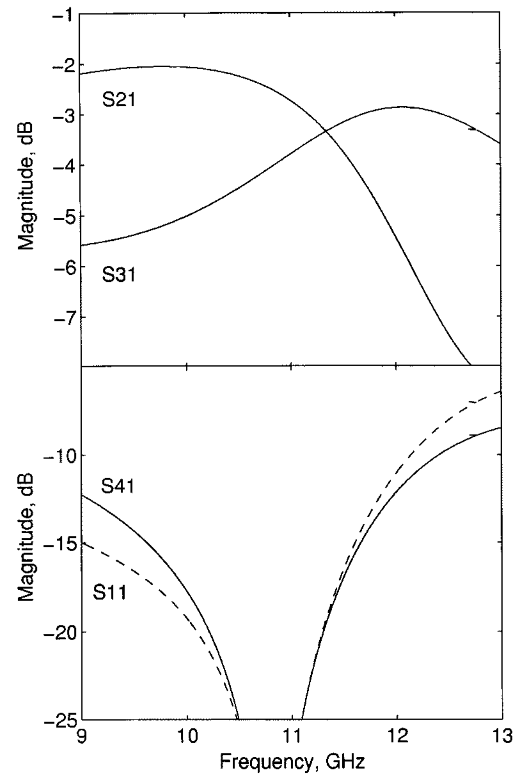

Fig. 4. Simulated frequency response of the MMIC unequal line-length branch-line coupler of Fig. 3.

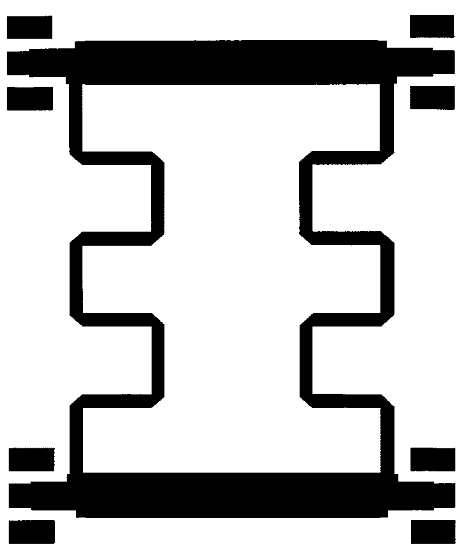

Fig. 3. Layout of the MMIC unequal line-length branch-line coupler designed on GaAs substrate using GEC Marconi F-20 process foundry rules. $f_0 = 11$ GHz, $Z_1 = 40\ \Omega, Z_2 = 66.67\ \Omega, \theta_1 = 62.11°$, and $\theta_2 = 131.41°$. For through lines, $L = 1634\ \mu$m and $W = 240\ \mu$m. For branch lines, $L = 3458\ \mu$m and $W = 72\ \mu$m.

coming comparable with the line lengths at high frequencies [6] by enabling designs with higher characteristic impedances; hence, narrower linewidths.

Fig. 2 shows the comparison of the frequency response of a 3-dB unequal line-length branch-line coupler (the first design given above) with that of the corresponding conventional coupler with quarter-wave-line lengths. It is observed that the branch-line couplers employing quarter-wave-long lines give wider bandwidths than the alternative design presented in this paper does. This difference becomes more pronounced as the lengths deviate further from the quarter-wave.

As another example, a MMIC unequal line-length branch-line hybrid is considered. Fig. 3 shows the layout of the hybrid, which is designed on a GaAs substrate using GEC Marconi F-20 process foundry rules. In this process, the minimum realizable characteristic impedance is 40 Ω and, hence, a conventional hybrid, which requires $Z_1 = 35.35\ \Omega$, cannot be realized. Fig. 4 shows the simulated frequency response of this unequal line-length MMIC hybrid. The effects of the junctions and corners are taken into account. The shift in the center frequency and the finite reduction in the return losses are assumed to be due to the losses of the substrate.

IV. Conclusions

Exact design equations for alternative branch-line couplers, which use unequal line lengths have been presented in this paper. Out of four variables, that are two lengths and two characteristic impedances, one can be chosen arbitrarily to facilitate the realizability problems. The analysis of this type of branch-line coupler is reduced to three equations from which a hybrid with any power division ratio can be designed.

In the design examples provided, it is shown that higher impedance levels can be used for the branches to overcome the difficulty of the linewidths becoming excessive, particularly at higher microwave frequencies. As an example, an alternative design for a MMIC unequal line-length branch-line coupler is given where the conventional branch-line coupler having 35.35-Ω characteristic impedance could not be used because of the concerned MMIC foundry rules. In this example, a compact design occupying a relatively small chip area is realized.

The inherent characteristics of the unequal line-length branch-line coupler is the narrow-band operation compared to the commensurate line couplers. This feature becomes more noticeable when the impedance levels deviate further away from those of the conventional coupler or the lengths differ further from quarter-wave. This is an important drawback of the present coupler, and for wider bandwidths different methods, such as the optimization technique in [3], could be used.

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A Simple Equation for Analysis of Nonuniform Transmission Lines

Robert Nevels and Jeffrey Miller

***Abstract*—The solution to the telegrapher equations is often presented as a D'Alembert expression for the voltage in terms of the voltage at a previous time or for the current in terms of the current at a previous time. In this paper, we present a complete solution for the coupled set of transmission-line equations such that the voltage or current is in terms of both the previous time voltage and current amplitudes. The key features of these equations are: they require only the initial voltage and current amplitudes, positive- and negative-direction traveling waves do not have to be identified, they are valid on a nonuniform transmission line, and they are related to the frequency-domain $ABCD$-parameter equations and the D'Alembert expressions for coupled functions. A method is presented for evaluating this set of equations numerically and results are given for a transmission-line filter and for a transmission line with a nonuniform section.**

***Index Terms*—Distributed parameter circuits, transmission lines.**

I. INTRODUCTION

In many textbooks, e.g., [1], general real-time solutions to the transmission-line voltage and current wave equations

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \tag{1a}$$

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} \tag{1b}$$

for a lossless line are given, respectively, by the D'Alembert solutions

$$V(z,t) = V^+\left(t-\frac{z}{v}\right) + V^-\left(t+\frac{z}{v}\right) \tag{2a}$$

$$I(z,t) = I^+\left(t-\frac{z}{v}\right) + I^-\left(t+\frac{z}{v}\right) \tag{2b}$$

where L and C are the transmission-line inductance and capacitance per unit length and $v = 1/\sqrt{LC}$ is the velocity of the line voltage and current waves. The interpretation of (2a) is that the voltage at a particular time and position, i.e., t and z, respectively, on a transmission line is exactly equal to the sum of the forward (V^+) and reflected (V^-) traveling voltage waves that existed at a time $t' = t - z/v$, at the respective points $z' = z \mp vt$. The expression for the current (2b) can be interpreted in the same way. Although useful, implicit in (2) are the assumptions that the transmission line is uniform and that the incident and reflected voltage and current wave functions are known.

Below, we present a simple powerful alternative set of transmission-line equations in which it is not necessary to specify the direction of travel of the initial input signal or even to distinguish the incident from the reflected waves. In addition, these equations are valid when the transmission line is nonuniform. Nonuniform transmission lines are of particular importance in modern-day microwave circuits. For example, tapered lines and concatenated lines with different characteristic impedances appear both in analog and digital circuits as impedance-matching devices and filters [2], [3].

II. ANALYSIS

The transmission-line equations

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \tag{3a}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \tag{3b}$$

for a two-port lossless nonuniform transmission line can be cast in the form of a single-vector equation

$$\frac{\partial \mathbf{F}}{\partial t} = \bar{\mathbf{S}}\mathbf{F} \tag{4}$$

where the voltage–current vector F and operator matrix $\bar{\mathbf{S}}$ are defined by

$$\mathbf{F} = [V \quad I]^T \tag{5}$$

$$\bar{\mathbf{S}} = -\begin{bmatrix} 0 & \frac{1}{C}\frac{\partial}{\partial z} \\ \frac{1}{L}\frac{\partial}{\partial z} & 0 \end{bmatrix}. \tag{6}$$

In the above equations, the distributed inductance (L) and capacitance (C) are each functions of position z, as are the current (I) and voltage (V), which are also functions of time. A solution to (4) can be found by first finding the propagator matrix $\overline{\mathbf{K}}$ that satisfies [4]

$$\frac{\partial \overline{\mathbf{K}}}{\partial t} = \overline{\mathbf{SK}} \tag{7}$$

with the initial condition

$$\lim_{t\to 0} \overline{\mathbf{K}} = \bar{\mathbf{I}}\delta\left(z - z'\right) \tag{8}$$

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The authors are with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843 USA.

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